H. Lee - 27/10/11 - continued from last week

Recall: $L_0 = V,\ L_1,\ L_2 \simeq \tau_V(L_1) \subseteq M$

Constructed: $c \in CF(L_1, L_2)$ $\qquad \mu^1(c) = 0$

$k : CF(L_0, L_1) \to CF(L_0, L_2) \qquad \mu^2(c, \cdot) + k(\mu^1(\cdot)) + \mu^1(k(\cdot)) = 0.$

Now we wish to upgrade this to a contraction on the associated Yoneda modules

ie: consider an auxiliary exact Lagrangian Q; fix Floer & perturb data

Construct:

$$d : CF(Q, L_1) \longrightarrow CF(Q, L_2)$$
$$l : CF(L_0, L_1) \otimes CF(Q, L_0) \to CF(Q, L_2)$$
$$\mu^1(d(\cdot)) + d(\mu^1(\cdot)) = 0$$
$$d(\mu^2(\cdot,\cdot)) + \mu^1(l(\cdot,\cdot)) + l(\mu^1(\cdot),\cdot) + l(\cdot, \mu^1(\cdot)) = 0$$

This means that

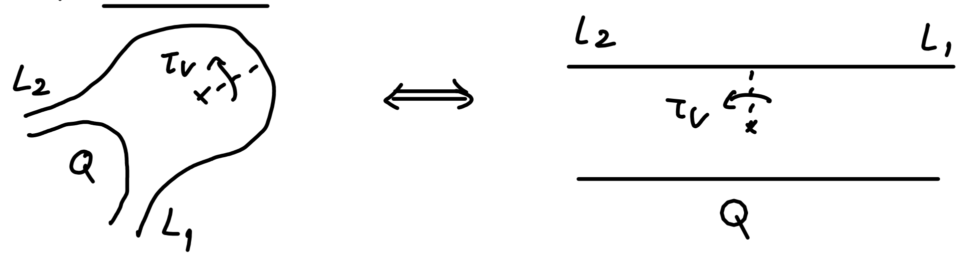
$$D = \begin{tikzcd} \big(CF(L_0,L_1) \otimes CF(Q,L_0)\big) \arrow[r, bend right, "\mu^2"'] \arrow[rr, bend right=50, "l"'] & \oplus\ CF(Q,L_1) \arrow[r, bend right, "d"'] & \oplus\ CF(Q,L_2) \end{tikzcd}$$

is a <u>twisted complex</u>, ie. $\partial^2 = 0$ where $\partial = \begin{pmatrix} \mu^1 & & \\ \mu^2 & \mu^1 & \\ l & d & \mu^1 \end{pmatrix}$

<u>Thm</u> (Seidel): The complex (D, ∂) is <u>acyclic</u>.

Constructing d and l:

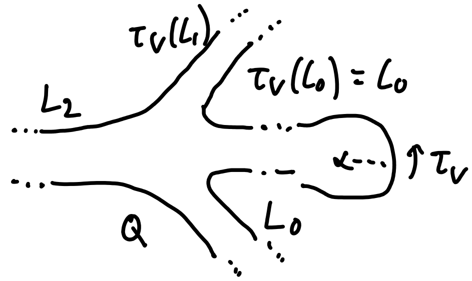
- <u>d counts</u> (index 0) <u>sections of</u>:



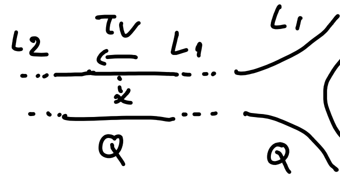
Show $\mu^1 \circ d + d \circ \mu^1 = 0$ by considering ∂ of 1-dim'l $\mathcal{M}$ of index 1 sections (= break index 1 strips).

- l <u>counts</u> index -1 sections of family of fibrations interpolating b/w

$E^{\ell,0}$
$\downarrow$
$S^{\ell,0}$



and



$E^{\ell,1}$
$\downarrow$
$S^{\ell,1}$

$C\Phi(E^{\ell,0}/S^{\ell,0}) = 0$
(vanishing thm)

$C\Phi(E^{\ell,1}/S^{\ell,1}) = d(\mu^2(\cdot,\cdot))$

$\bigsqcup_{r\in[0,1]} \mathcal{M}^0(E^{\ell,r}/S^{\ell,r})$ = 1-dim'l mfld with boundary

- $\mathcal{M}^0(E^{\ell,0}/S^{\ell,0})$
- $\mathcal{M}^0(E^{\ell,1}/S^{\ell,1})$
- $\bigsqcup_{r\in(0,1)} \left(\mathcal{M}^{-1}(E^{\ell,r}/S^{\ell,r})\right) \times (\text{strips})$

$\Rightarrow$ properties follow.

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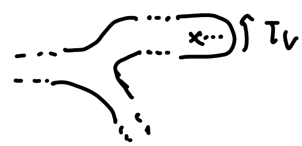
* <u>Moreover</u>: if $(\tilde{d},\tilde{\ell})$ arise from different construction choices, then can still show $(\tilde{d},\tilde{\ell})$ is homotopic to (d,ℓ).

This applies in particular to :
$$\begin{cases} \tilde{d} = \mu^2(c,\cdot), \\ \tilde{\ell} = \mu^2(k(\cdot),\cdot) + \mu^3(c,\cdot,\cdot) \end{cases}$$

Since $\tilde{d} = \mu^2(c,\cdot)$ counts sections of



= degenerate limit of contr. of d above

& similarly for $\tilde{\ell}$, count sections of

= —— " —— of ℓ.

This gives us <u>exact triangle</u> : indeed $\tilde{d} = \mu^2(c,\cdot)$
$\tilde{\ell} = \mu^2(k(\cdot),\cdot) + \mu^3(c,\cdot,\cdot)$

give us $\forall Q$,

$$\begin{tikzcd} CF(L_0,L_1)\otimes CF(Q,L_0) \arrow[r, "\mu^2"'] \arrow[rr, bend right, "\tilde{\ell}"'] & CF(Q,L_1) \arrow[r, "\tilde{d}"'] & CF(Q,L_2) \end{tikzcd}$$

acyclic complex

& this complex behaves naturally wrt. Q.

In fact, these complexes assemble into a twisted complex of Yoneda modules

$$L_i \underset{\text{Yoneda}}{\longmapsto} \mathcal{L}_i = CF(-, L_i) \qquad i = 0,1,2.$$

$$\begin{tikzcd} CF(L_0,L_1) \otimes \mathcal{L}_0 \arrow[r, "t"', "\text{tautological}"] \arrow[rr, bend right, "k[-1]"'] & \mathcal{L}_1 \arrow[r, "c"] & \mathcal{L}_2 \end{tikzcd}$$

So now let

$$\begin{tikzcd} K = \mathrm{Cone}\left(CF(L_0,L_1) \otimes \mathcal{L}_0 \xrightarrow{t} \mathcal{L}_1\right) = \left(\left(CF(L_0,L_1) \otimes \mathcal{L}_0\right) \oplus \mathcal{L}_1 , \begin{pmatrix} \mu^1 & 0 \\ t & \mu^1 \end{pmatrix} \right) \arrow[d, "{(c,k)}"'] \\ \mathcal{L}_2 \end{tikzcd}$$

- ⭑ (c,k) is a closed morphism of twisted complexes
- ⭑ it is a quasi-iso. (since complex $K \xrightarrow[(c,k)]{} \mathcal{L}_2$ acyclic) (hence invertible).

By def. we have exact triangles

$$\begin{tikzcd} CF(L_0,L_1) \otimes \mathcal{L}_0 \arrow[rr, "t"] & & \mathcal{L}_1 \arrow[dl] \\ & K \arrow[ul] & \end{tikzcd}$$

and similarly via (c,k),

$$\begin{tikzcd} CF(L_0,L_1) \otimes \mathcal{L}_0 \arrow[rr] & & \mathcal{L}_1 \arrow[dl, "c"] \\ & \mathcal{L}_2 \arrow[ul, "T"] & \end{tikzcd}$$